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Educating Cadets Competencies using the Special Relativity Theory

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Abstract

The cadets' competencies in the contemporary geopolitical situation cover a broad spectrum of skills, abilities, knowledge and attitudes. Among other things, it is also knowledge in the field of physics. The standard approach to the teaching of special relativity is about presenting the postulates, then showing the transformation relations between rest and moving frames, following with formulae for the adding of the velocities, dilation of the time and contraction of the length. Here we present another approach to the same topic, which is built on the concept of spacetime, especially on the spacetime diagrams, and with detailed focus on the 'paradoxes', in particular the twin paradox and the ladder paradox. The text is based on author's own experiences with presenting these concepts to the students.

KEY WORDS: *cadets competencies, special theory of relativity, spacetime diagram, twin paradox, ladder paradox*

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1. Introduction

The future world will be volatile, uncertain, complex and ambiguous, but the physical laws are immutable. To estimate how the society will change over time and which knowledge cadets will need to not just survive but also to be successful is very difficult. Therefore, it is important to keep an attention on their competencies, to use the connection of knowledge from various educational areas into broader units with the aim of creating a more comprehensive view of mathematical, natural, social and cultural phenomena.

In the standard physical textbook for engineers which has a chapter on the special relativity [1], [2], there is no notation of spacetime diagrams. Even in the famous textbook on special relativity for theoretical physicists [3], there is just one spacetime diagram with the light cone. But the spacetime diagram approach to the special relativity can be beneficial for the students as the diagrams (and the pictures in general) are easily understandable than the complicated physical formulae.

As a lecture on the special relativity is a part of the syllabus at University of Defence, we present here an alternative way to special relativity to the military students. In general, the teaching of the military students has its own specifics, for the case of University of Defence see [4], [5]. The following text does not have a purpose to replace the standard approach, but more likely to extend it. At least the notion of spacetime diagrams from the first section of this text can be incorporated into teaching, because it naturally expands and supplements the concepts of dilation of time and contraction of the length, which are taught in the standard approach. The second and third section about 'paradoxes' are quite more involved and can be used for the students as the voluntary home assignments.

The following text is based on tutorials which was taught by the author at Masaryk University for six years. The text is enlarged and simplified for the usage at the University of Defence.

2. A first look on spacetime with spacetime diagrams

A spacetime is a collection of the classical three-dimensional space with one-dimension time direction. In this text we will work with just two-dimensional flat spacetime, also known as two-dimensional Minkowski space. A spacetime diagram is a plot with one temporal axis, while the other axis is spatial as usual. For the convenience the temporal axis has label ct, therefore the both axes have the same dimension, as the second axis has label x. The spacetime diagram is always connected with an observer in the rest. While at the purely spatial diagrams, the observer in the rest stays in the one point, which we can choose without the loss of generality as an origin, the spacetime diagram is in some sense 'dynamical'. As one axis is temporal, the observer in the rest can't stay in one point, instead his/her movement through the spacetime diagram is described by an equation

$$x = 0.$$

That means that his/her spatial coordinate is fixed, and his/her trajectory through the spacetime diagram (which is called a worldline) is a vertical straight line, which coincidences with the axis ct.

Let's assume that the observer in the rest (we call her Alice from now) at the spacetime point $(ct, x) = (0,0)$ sends a light signal. The light is moving with the speed of light c, thus its worldline has an equation

$$x = \pm ct,$$

where the sign depends on the direction of the light signal, if the Alice sends it to the left or the right (see Figure 1). Let's assume also, that Alice meets another observer, called Bob, at the spacetime point $(ct, x) = (0,0)$. But Bob has non-zero velocity with respect to Alice, he has velocity $v_B = \frac{c}{2}$. Therefore his worldline is

$$x = \frac{1}{2}ct.$$

See the Figure 1 for the picture of the situation.

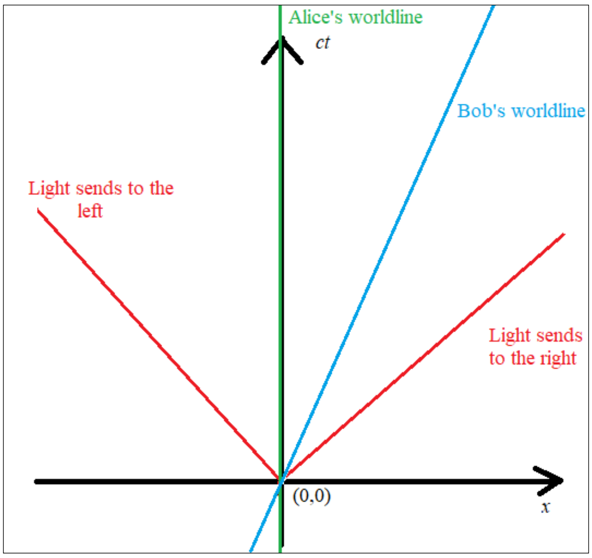

Fig. 1. Alice is in the rest. She sends light signal both ways and also meets Bob at the spacetime point (0,0).

The main idea of the relativity is the following: how does Bob see this situation? Let's denote his temporal axis as ct' and his spatial axis x'. At his frame, he is in the rest, that means his worldline has an equation

$$x' = 0.$$

From his point of view, he meets Alice at the spacetime point $(ct', x') = (0,0)$, while Alice has the velocity $v_A = -\frac{c}{2}$ (the magnitudes of velocities are the same, but they have opposite directions), therefore Alice has the worldline

$$x' = -\frac{1}{2}ct',$$

at Bob's frame. And what about the light signal, which Alice sends? One of the postulates of the special relativity is, that the light has the same speed in the all inertial frames. Thus, the equations for the light worldlines are

$$x' = \pm ct'.$$

The situation from the Bob's point of view is depicted on the Figure 2. The transformation from the Alice's frame to the Bob's frame and vice versa is called Lorentz transformation [3]

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \gamma & \frac{v_A}{c}\gamma \\ \frac{v_A}{c}\gamma & \gamma \end{bmatrix} \cdot \begin{bmatrix} ct \\ x \end{bmatrix}, \qquad \begin{bmatrix} ct \\ x \end{bmatrix} = \begin{bmatrix} \gamma & \frac{v_B}{c}\gamma \\ \frac{v_B}{c}\gamma & \gamma \end{bmatrix} \cdot \begin{bmatrix} ct' \\ x' \end{bmatrix}, \tag{1}$$

where γ is so called Lorentz factor, which is defined as

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}},$$

with $v = -v_A = v_B$. The matrices in the equation (1) are inversed to each other.

One of the key concepts in the special relativity is non-existence of the absolute time. Instead there is a notion of the proper time and the coordinate time. The proper time is different for each observer, as each observer has his/her own clock (or own cell phone). In the example above with Alice and Bob, the proper time of the Alice is denoted t, and proper time of the Bob is denoted t'. The coordinate time is on the other hand always connected to the rest frame. On the Figure 1 the coordinate time is t, which is Alice proper time.

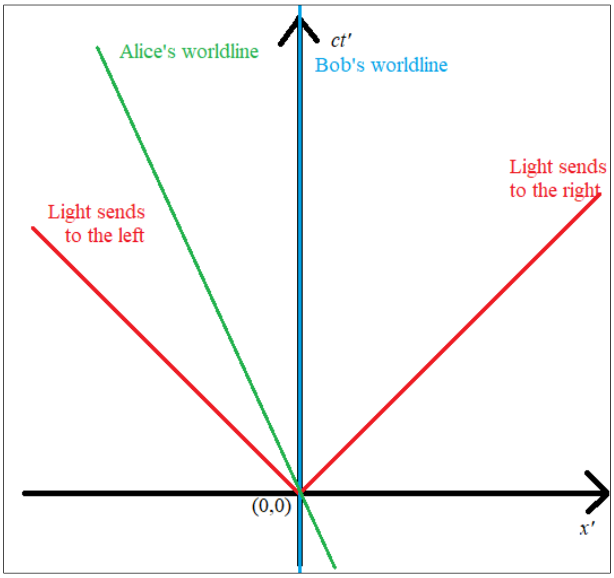

Fig. 2. Bob is in the rest. He meets Alice at the spacetime point (0,0), where she emits two light signals.

On the Figure 2 the coordinate time is t', which is Bob's proper time. The coordinate time serves as universal time for the frame, a time which all observes agree to use, because the communication is easier[3]. The proper times are different in general, as is obvious from the equation (1). If we are interested in the time intervals, i.e. how much time passes in the one frame in comparison to the second frame, we discover the dilation of the time. Let's say that Bob starts a stopwatch in time t_1' and stops it in time t_2' . Therefore, the time on the stopwatch is $\Delta t' = t_2' - t_1'$. But at his rest frame, he doesn't move, he still stays at the position $x' = 0$, thus $\Delta x' = 0$. The question is: how much does time pass in the Alice's frame? The answer is given by the equation (1)

$$\begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix} = \begin{bmatrix} \gamma & \frac{v}{c}\gamma \\ \frac{v}{c}\gamma & \gamma \end{bmatrix} \cdot \begin{bmatrix} c\Delta t' \\ 0 \end{bmatrix},$$

and by the matrix multiplication we find out that

$$\Delta t = \gamma \Delta t'. \tag{2}$$

This is the famous time dilation. As the γ is always greater or equal to 1 the following holds

$$\Delta t \geq \Delta t'.$$

Or in the words: the Alice always measures the longer time interval in her frame than Bob in his frame. Or in another words: the moving observer get older more slowly with respect to the observe in the rest. These statements have some conceptual loopholes and we get back to them in the next section. The dilation of time is the effect of special relativity on the

[3] The similar real-world situation is with UTC, the time zones times and the local times. There is no special relativity, but as each place has the different time (due to the rotation of the Earth and the Sun), it is easier the agree on one universal time, UTC, or 24 time zones. Otherwise each two locations on different meridians would have different times, and at least public transportation would be almost impossible.

temporal axis, the effect of the special relativity on the spatial axis, called the length contraction, is more involved and can be left to the students as home assignment[4].

On the Figure 1 and Figure 2 we can notice a structure which doesn't change from one frame to another, the light rays. This should not be surprising as light has the same velocity in every frame. Moreover, this structure of the light rays, called light-cone, can be used to discover the causal structure of the spacetime.

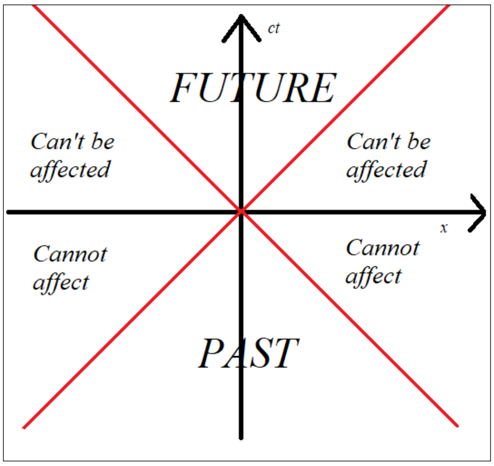

Fig. 3. Light-cone structure of the spacetime.

On Figure 3 there is a spacetime diagram with a fixed observer at the origin of the spacetime, i.e. spacetime point $(ct, x) = (0,0)$. As the speed of light is the maximum speed, the information to the observer in origin could get just from part of the lower half of the diagram, which is between two light rays. This is the 'past', the part of the spacetime which can have an influence on the observer at the origin. The edges of the 'past' are light rays, all other observes which can meet our observer have to be between these light rays, as they speed is smaller than the speed of light. On the other hand, the upper part of the diagram between light rays is the 'future' to the observe at the origin. This part of the diagram can be influenced by the observer in the origin. His/her own worldline, due to being in the rest frame, is the axis ct, but he/she can throw something, or send light signal to affect this part of the spacetime. The last part of the spacetime is on the left and on the right. This part of the space can't be affected by the observer in the origin or cannot have an influence on the observer in the origin. And the reason why is obvious. Because the information has to travel in larger speed that the speed of light. One last note to the light-cone structure, which we have to have in mind, is that the Figure 3 is frozen in one particular time. The spacetime diagram is dynamical, therefore the observer in the origin should move on his/her worldline, i.e. axis ct. As the light-cone is defined in each point of the worldline, it moves with the observer, and each point has another 'past' and 'future'. This brings us to the propagation of the information in the spacetime.

[4] In our example with Alice and Bob, we can formulate the problem as follows. Alice measures the length of Bob's trajectory. In her frame the trajectory is Δx, in Bob's frame the trajectory is $\Delta x'$. The crucial point here is, that the measurement has to be done in the one time, and as Alice is the one, who measures, this means $\Delta t = 0$. Plugging these to equation (1), we can solve the equation for $c\Delta t'$, and plugging this to the second equation we find out the contraction of the length $\Delta x = \Delta x'/\gamma$.

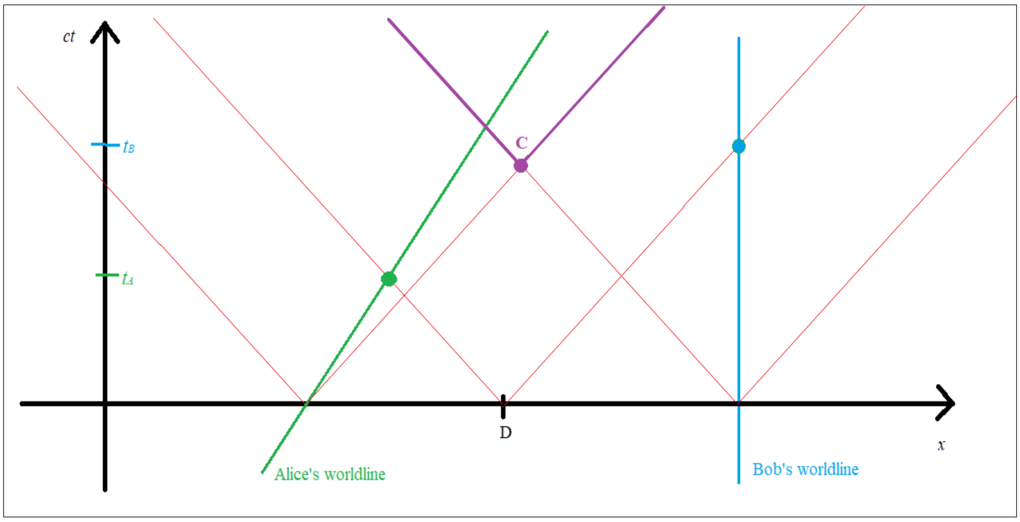

Fig. 4. Propagation of the information in the spacetime diagram.

The information in the spacetime can propagate in various speed, but the maximum speed of propagation is speed of light. As all information could be encoded into binary code, which could be propagate for example by lasers (laser emits a signal is 'one', laser doesn't emit a signal is 'zero'), we will work just with information which propagates with speed of light. On the Figure 4, there are again two observers Alice and Bob. Bob has velocity with respect to the rest frame $v_B = 0$, Alice has velocity $v_A = c/2$. The axes are ct and x, which are different to Alice's rest frame (because she has non-zero velocity) and also to Bob's rest frame (as his worldline doesn't cross the origin of the rest frame). There are also drawn light-cones in the coordinate time $t = 0$. Therefore the first spacetime point which can be affected by both Alice and Bob at time $t = 0$, is the spacetime point C. And the whole part of the spacetime which can be affected by both Alice and Bob is a cone with vertex C. Let's say that in time $t = 0$, something terrible happened (like an explosion of a supernova) in the spacetime point D (the point is in the middle between of Alice and Bob). When they get an information about this event? As we said, the information propagates with speed of light, therefore we draw another light-cone from the spacetime point D. And the intersections between this light-cone and Alice's and Bob's worldline give us the exact moments, when both of them get the information about the explosion. It should not be surprising that $t_A < t_B$, as the Alice moves towards the explosion. But this is one of the crucial ideas in special relativity, that information is not instant, it takes a time to get the information. The concept which could leads to some paradoxical situations as we will see in the ladder paradox.

The last remark in this section is about the second postulate of the special relativity, that the physics in all inertial frames is the same. How we can easily see in the spacetime diagram, that the observe is not in the inertial frame? The answer is very straightforward. The difference between inertial and non-inertial frame is an acceleration. And if an observer has non-zero acceleration, his/her worldline is not a straight line, but rather a curved line, or there is a tip on the line.

3. The Twin Paradox

The statement of the twin paradox is the following: *Consider two twins, first stays on Earth, second travels with the speed, close to the speed of light, to the space. After a time, the second twin turns back home. When they meet each other again on the Earth, the second twin is older, because of the dilation of time. But from the frame of the reference of the second twin, the twin, which stays on Earth, should be older, because the second twin was at his/her frame in the rest and the first twin has non-zero velocity.* And that's the paradox. Except for one thing. There is no paradox, just weakly understood basics of special relativity. If we draw the spacetime diagram for this situation, which is on the Figure 5, we see that the second twin feels the acceleration at the turning point. There is a tip on the worldline of the second twin. Therefore, the second twin is not an inertial observer and thus, the special relativity is not enough to describe this situation. This implies that this can't be a paradox within the theory of special relativity, as it contradicts the one of the postulates of the theory. In fact, if you want to describe this situation, you need general theory of relativity, i.e. theory of non-inertial frames[5].

[5] Also, you need to remove an infinite acceleration at the starting and turning point, which is unphysical. But if you formulate the twin paradox as follows: First twin stays on the Earth, second twin accelerates with acceleration $9.81\ \mathrm{m} \cdot \mathrm{s}^{-2}$ for one year, then accelerates with the same acceleration but with opposite direction for two years and finally accelerates with the same acceleration but with the origin direction for last year. After all, the second twin is back on the Earth and the journey takes four years in his/her reference frame. But from the point of the twin on the Earth, the journey takes 4,78 years.

But we can slightly change the settings of the situation to understand more about the spacetime. Let's assume that two twins, we call them Cedric and Dedrick, are totally identical. They have the common medical history and future, i.e. if one of them has a heart attack, the other also has a heart attack. Cedrick stays on the Earth, Dedrick travels to the stars with speed

$$v = \frac{\sqrt{3}}{2}c.$$

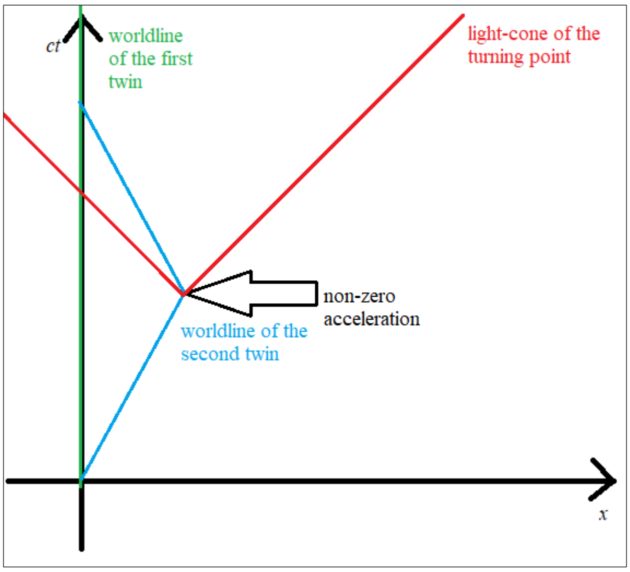

Fig. 5. The twin paradox. The arrow points on the turning point with non-zero acceleration. In the highlighted light cone, there is the part of the spacetime, where the special relativity is non-enough to describe the situation.

Two years after Dedrick's departure, Cedrick has a heart attack. From the dilation of time, equation (2), we can calculate the Dedrick's proper time. Two years are in the Cedrick's frame of reference, i.e. $\Delta t = 2$ years. We know the speed; therefore, we can get the time which has passed in Dedrick's frame of reference, after the calculation we find out that it is $\Delta t' = 1$ year. Thus, he has still one year until he is going to have the heart attack. So, the question is the following, can we send this information to him before the heart attack and possibly save his life? For the answer we use the spacetime diagram. On Figure 6 there is Cedrick's worldline C, Dedrick's worldline D and after two years, the light signal S is emitted from the Cedrick's worldline. The light signal S crosses the Dedrick's worldline in the spacetime point P.

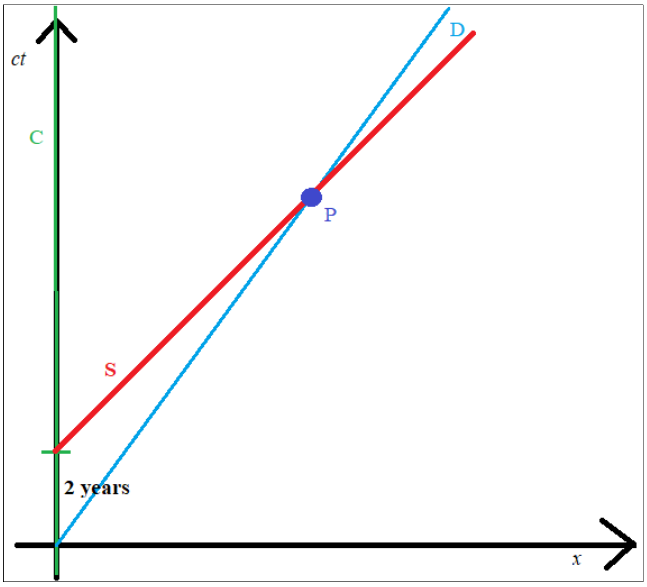

Fig. 6. The spacetime diagram of Cedrick and Dedrick.

This is the moment, when Dedrick gets the information about Cedrick's heart attack. Our task is to find out the spacetime coordinates of this point. And now we will see the power of the spacetime diagrams, because we just need to calculate the intersection of two straight lines. It reduces the abstract physics to the level of high school geometry! Let's calculate. We can think about the worldlines as functions of x. Thus, the Dedrick worldline has an equation

$$\text{D: } ct = \frac{2}{\sqrt{3}}x.$$

The light signal S travels with speed of light and has an equation

$$\text{S: } ct = x + 2.$$

We have two equations with two unknows, therefore we can solve this system of equations. The solution is the spacetime coordinates of the point P in Cedrick's reference frame. The spacetime point P is

$$\text{P} = (ct, x) = (14.93\, c \cdot \text{years}, 12.93\,\text{ly}).$$

The signal crosses the Dedrick's worldline after 12.93 years (14.93 years after the Dedrick's departure) in the distance of 12.93 light years. But keep in mind, that this is calculated in the Cedrick's frame. We have to transform these numbers to the Dedrick's frame. We plug the elapsed time into (2), and we find out, that in the Dedrick's reference frame 7.465 years has passed, therefore, Dedrick's heart attack was 5.465 years ago, and we can't save him. On the other hand, we show that the causality in the special relativity is preserved. The consequence follows the cause, and not vice versa.

One last note on this example. What about numbers that we got? Derick's spaceship is 12.93 light years away from the Earth, but on the spaceship just 7.465 years has passed. That seems that the spaceship flies faster than light? This question can be left as the home assignment for students. The answer is the contraction of the length. The distance 12.93 ly is measured in Cedrick's reference frame. But for Dedrick this distance is moving, because in his frame, he is in the rest, and the rest of the universe is moving. Therefore, he measures the moving length and due to contraction of the length, the length is smaller. We can plug the numbers into formula in the Footnote 2 to see, that in Dedrick's frame, Dedrick travels just 6.465 ly. And it makes sense, he has velocity $\sqrt{3}/2 \cdot c$, he flies for 7.465 years, and if we multiply these two numbers, we get again 6.465 ly. Everything works perfectly.

4. The Ladder Paradox

The ladder paradox shows how our intuition fails in the special relativity problems. It can be formulated as follows: *A person holds a ladder, which is* 2.1 m *long, in front of them. This person has the velocity*

$$v = \frac{\sqrt{3}}{2}c,$$

and runs into a room, which is only 1 m *long. But the person is capable of closing the door when enters the room. Explain why?* On the first sight this is non-sensical situation, you can't fit 2.1 m long ladder into 1 m long room. On the second sight, and with a little knowledge about special relativity, it still not makes sense. The length of the ladder, as the ladder is moving, is contracted. Plugging velocity and length into the formula from Footnote 2 we find out, that in the room's frame of reference, the ladder has just 1.05 m. The ladder is shorter but not enough to fit into room. For the whole answer we need spacetime diagram. We call the person holding the ladder Usain and we work in the reference frame which is connected to the room. The situation is on the Figure 7.

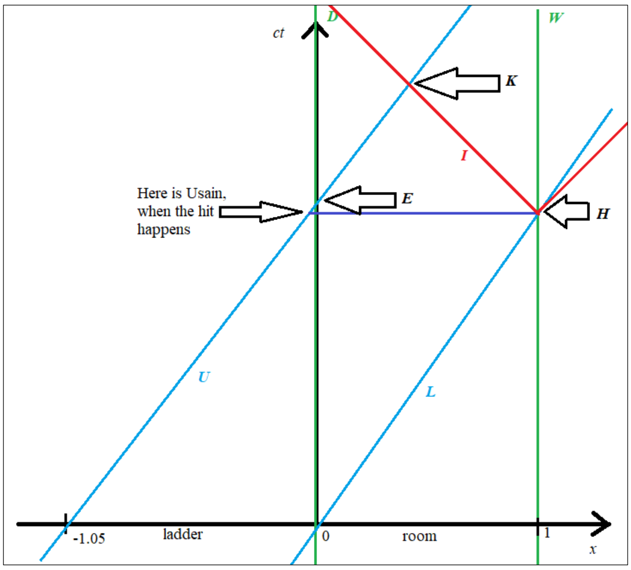

Fig. 7. The spacetime diagram for the ladder paradox.

First look at the axis x, there is the ladder in the moment, when the first part of the ladder enters the room. The ladder is between points -1.05 and 0. Usain is at the point -1.05. On the other part of the diagram, there is the room. At the point 0, there are doors, at the point 1, there is a wall. Because we work in the room's reference frame, the ladder is contracted. These all things are in one moment of time, when $t = 0$. Now we start to draw worldlines. First the easy ones. The room is in its rest frame, it doesn't move, therefore its door has the worldline

$$D: x = 0,$$

and the wall has the worldline

$$W: x = 1.$$

The ladder is moving with Usain's velocity, so the top part of the ladder has the worldline

$$L: ct = \frac{2}{\sqrt{3}}x.$$

The slope of the Usain's worldline is the same, but he is shifted to the point -1.05 on the axis x, therefore, to get his worldline we first write general worldline with the slope $2/\sqrt{3}$

$$U: ct = \frac{2}{\sqrt{3}}x + A,$$

where the A is a constant, which we obtain by plugging the spacetime point $(ct, x) = (0, -1.05)$ into the worldline. Therefore, Usain's worldline is

$$U: ct = \frac{2}{\sqrt{3}}x + \frac{2.1}{\sqrt{3}}.$$

Now we scan the diagram from the bottom to the top to find some interesting events. We scan this diagram with respect to the time flow. On the bottom, there is the past, on the top, there is the future, thus the arrow of time is from bottom to the top. First of the interesting events is when the ladder hits the wall at the end of the room, we denote it with letter H. Where is Usain, when this hit happened? He is still 1.05 m to the left of the top of the ladder, so in front of the doors. But the information about the hit propagates through the spacetime with speed of light. It is not instant. So, we draw a light-cone of the hit and we denote the left light ray as I. We continue with the scanning of the diagram, Usain follows his worldline and at the point E he enters the room. He can do it, as the information about the hit doesn't cross his worldline. He has no idea about the ladder hitting the wall and nothing stops him. He can even run deeper into the room, up to the point K, where the information about the hit crosses his worldline.

But the spacetime diagram is not a proof of Usain being able to close the door. We have to calculate the spacetime coordinates of the point K. This point is the intersection of worldline U and I. We already have the equation of worldline U and for the worldline I, we have to know the coordinates of the point H. The point H is the intersection of the worldlines W and L, both equations can be found above. It is easy to see, that the point H has the spacetime coordinates

$$(ct, x) = \left(\frac{2}{\sqrt{3}}, 1\right).$$

Back to the worldline I. The general form of this left-moving light ray worldline is

$$I: ct = -x + B,$$

where the minus sign in front of x means that it moves with speed of light to the left, and B is a constant whose value we gain by plugging the coordinates of the point H. Again after quick calculation we get

$$I: ct = -x + 1 + \frac{2}{\sqrt{3}}.$$

From the last calculation we obtain the desired coordinates of the point K. We have equations for both worldlines on which the point K lies. These are worldlines U and I. We can subtract these equations from each other to get

$$0 = \frac{2}{\sqrt{3}}x + x + \frac{2.1}{\sqrt{3}} - 1 - \frac{2}{\sqrt{3}}.$$

The solution is

$$x = \frac{\sqrt{3} - 0.1}{2 + \sqrt{3}} \approx 0.437.$$

This means that Usain is almost in the middle of the room when he gets the information about the hit! But wait, is not here the same problem as in the twin paradox? At the point H, when the ladder hits the wall, there is an acceleration. The ladder has to deaccelerate due to the hit. As we don't assume anything about the width of the wall, or material of wall and ladder, we don't know how much it deaccelerates, but it deaccelerates. Therefore, the top part of the ladder is a non-inertial observer and should not be possible to describe its movement with special relativity. And this is true, but just within the light-cone with the vertex H. In this part of the spacetime, there is an acceleration, therefore, the special relativity is not enough to describe the physics here. But the point E and even point K are both out of this light-cone, they are both in the 'can't be affected' part of the light-cone of the point H (compare the Figure 7 with Figure 3). This is the difference between ladder paradox and twin paradox. In the twin paradox, the return of the second twin on the Earth was inside the light-cone of the turning point, in its 'future' part (compare Figure 5 with Figure 3). But one thing has these paradoxes in common. Both are not paradoxes at all within special relativity. And for the full picture of these situation you need to incorporate the general relativity (and in the case of ladder paradox also material properties of the wall and latter).

The last remark about the ladder paradox is also a possible home assignment for the students. They can try to solve the ladder paradox not in the rest frame of the room, but in the Usain's rest frame. The situation in this reference frame is even more absurd. In his rest frame Usain stays at one position, holding the 2.1 m long ladder in front of him. The room is moving with the speed $v = \sqrt{3}/2 \cdot c$ towards him, and the room is just 0.5 m long, as it has to be contracted due its movement. But the result is the same, he is able to close the door. Therefore, from his point of view it looks like you can fit something 2.1 m long into 0.5 m long space! The calculations are similar to the presented ones, it's not hard nor easy, and if a student gets right answer, he/she understands the spacetime in right way.

1. Conclusions

We presented here spacetime diagram approach into special relativity. This approach is established on diagrams with one temporal and one spatial axis. In the realm of special relativity, all worldlines in these diagrams are straight lines, thus the problems in the special relativity are reduced to the level of high school geometry. Any curved line means, that the right physics to describe the situation is the general relativity. Therefore, another advantage of this approach is, that it can be easily generalized and it can be used as the entryway into the studying of the general relativity. Moreover, in the general relativity, there is a notion of another spacetime diagrams, which are called Penrose diagrams, and they are used to investigate the causal structure of the spacetimes.

The advantage of this approach is also its simplicity. For the students it is easier to think in the pictures than in the equations. As everything in the spacetime diagram can be drawn, the student can first draw everything and then think about the equations, which describe the worldlines in the diagram. From the point of view of a lecturer, this approach also easily opens up discussion with students about special relativity and even beyond, which can lead to another interesting topics.

A lecture based on this new approach was performed this year for the first time at University of Defence. It was an extra lecture based on the wish of the students, thus it is not in the syllabus of the subject and no compare tests were written. Also, the evaluation of the anonymous student survey is in the October of this year, therefore there are still no data objective data on this approach yet. All we have is a direct feedback from the students after the lecture, which was very positive.

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