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Drone Swarming and its Use in Minefield Laying Using Mathematical Methods

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Abstract

The work deals with the modelling of minefields using smaller swarms of different shapes. The aim of the work is to use mathematical methods to obtain clearer data on the possibilities of creating minefields using unmanned vehicles. On the basis of the minefield density parameters and the probabilities of hitting a mine according to the theory of at least once recurring phenomenon, possible variations of minefield laying are determined. The contribution of this work lies in presenting the real possibility of laying minefields in a completely new way. The calculations performed are identical to those performed for standard minefields.

KEY WORDS: *Drone, swarming, UAV, military, minefield, mines, minelaying, engineer support, density, probability*

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1. Introduction.

Significant technological advances in autonomous vehicles and drones are changing the dynamics of modern conflicts. The war in Ukraine has provided evidence of how this technology can change the course of combat. Even well-equipped militaries, such as the Russian Federation, face challenges posed by the deployment of autonomous unmanned vehicles. Their ability to carry out long-range attacks with minimal risk to their own forces gives a new dimension to military operations. [1]

However, drones are not just a tool of attack. They are also used to scout and track enemy movement and targets. Their ability to quickly and efficiently gather battlefield information allows command staffs to better navigate and make strategic decisions in real time [2]. In the area of mine warfare, autonomous assets are revolutionizing the field. Minefields built with unmanned assets enable faster and more flexible deployments. Analytical models and mathematical methods can be used to optimize mine placement and maximize the effectiveness of minefields, increasing the ability to resist enemy advances and defend strategic positions. [3]

While technological advances bring new possibilities for warfare, it is important to recognize their ethical and humanitarian implications. Care must be taken to protect civilians and minimize unwanted collateral damage. The use of autonomous means should be subject to firm ethical principles and legal frameworks in order to minimize the risk of uncontrolled attacks and negligence in military operations. [1]

2. Significance of minefields and current methods of their establishment

In contemporary conflicts, mines have always been deployed in areas of anticipated enemy movement. Although mines are relatively simple devices, their cheap production, high efficiency and mass proliferation make them a problem not only for the attacking troops, but also for troops operating within the area, unfortunately, for civilians, as civilian casualties have already exceeded 1 000 since the beginning of the war in Ukraine. [4], [5] To inflict casualties on the enemy by merely concealing a prepared mine or explosive charge below ground level is always advantageous to the defender, as it inflicts casualties on the enemy with only a small load. The prevalence and relative popularity of mines is evidenced by the accompanying illustration (e.g. Fig.1), which graphically shows the area potentially mined.

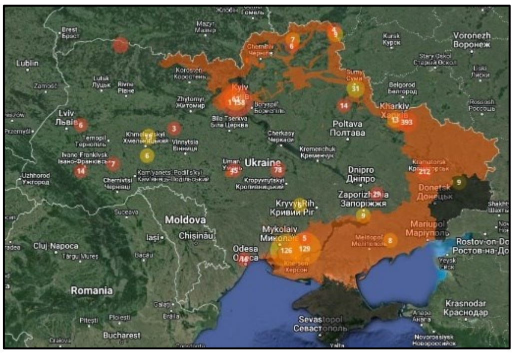


Fig.1. Undermined areas in Ukraine [4]

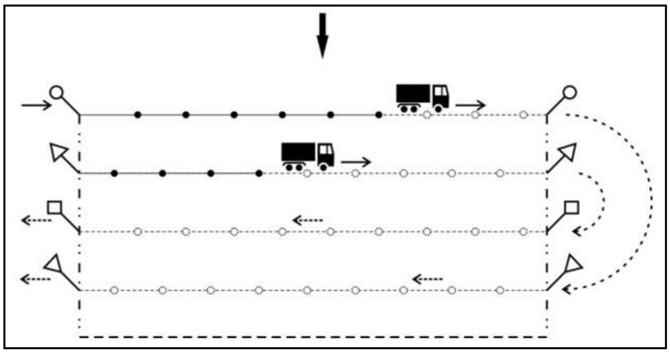


Fig. 2. Laying minefields using vehicles [7]

The data is taken as of February 2024. It is estimated that an area of 156 000 km^2 is undermined. This represents roughly 25% of the area of Ukraine and, for a better idea, almost three times the area of Croatia or twice the area of the Czech Republic. Nevertheless, more than 6 000 000 inhabitants remain in the designated areas and therefore increasing numbers of civilian casualties can be expected. [6]

In modern armies, where engineer units are also active, standardised methods for laying individual mines and large minefields have been established over time. Current mine laying methods are divided into arranged and scattered methods. It is this factor that will play an important role in the future. A minefield is considered to be an arranged minefield when individual mines are laid in a grid, checkerboard, or other shape that is determined in some way. Thus, the minefield is systematically laid, and all mines that have been placed in the minefield are traceable by means of a record and drawing of the minefield. An arranged minefield can be created by vehicles (e.g. Fig. 2) or by military personnel. In an arranged minefield, individual mines can be embedded in the ground to create an embedded minefield. Scatter minefields, on the other hand, are very non-deterministic. It is a method of minelaying in which proxies (rockets, helicopters, mortars) are used to barrage and block large areas in a very rapid time. It is the speed of minelaying and the difficulty of detection that is one of the serious threats to the assault troops, but also to the population. Landmines placed by a scatterable system are mostly laid on the surface of the ground (unless one of the landmines falls from a great height into softer soil). [7], [8]

The scattered method is characterised by irregular minefield edges. This is due to the unsystematic deposition caused by the means that can be characterized as the totality of the engineer, artillery, rocket launcher, and aerial systems used to bring mines to the mined area and to scatter them on the terrain. Scatterable minefields are located and their resulting shape can only be inferred from the characteristics of the carrier's trajectory. It is reported that 50% of all mines in a scattered minefield (e.g. Fig. 3) are located in ¼ of the area [7]. As an example of the scattered method, one can mention the Rosomacha minelaying kit (e.g. Fig. 4), which is a newly introduced quadrupedal vehicle to the Armed Forces of the Russian Federation, carrying SKM-A (Specialnyj komplekt minirovanija) throwers in the rear. Each launcher is fitted with four cartridges. This quadricycle is capable of dispersing small mines, identical to those used for dropping from helicopters or other self-propelled vehicles of the UMZ series. [9]

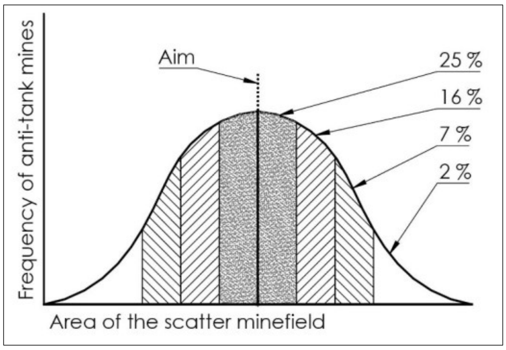


Fig. 3. Placement of mines in a minefield established in a dispersal manner [7]

Fig. 4. Minelaying off-road quad Rosomacha [9]

3. Drone swarming and its use in minefield

A swarm or fleet of Unmanned Aerial Vehicles (UAVs) consists of aerial robots, commonly known as drones, that collaborate to fulfill a particular objective. Each drone in the swarm is powered by a set number of rotors, allowing it to perform vertical take-offs, landings, and hovering maneuvers (VTOL). Drone flight can be controlled either manually via remote control or autonomously through onboard processors. Drones are frequently utilized for military purposes. Affordable drone swarms present a valuable opportunity for groundbreaking research and future commercial ventures that will benefit people in their daily activities and professional tasks. [10]

The concept of drone swarms – clusters of small, affordable drones that can collectively overcome enemy defences – is gaining traction. Equipped with weapons, even small drones can be highly destructive, especially in large numbers. These swarms, which can be remote-controlled, autonomous, or accompany other military assets, leverage advancements in artificial intelligence and drone miniaturization to multiply their effectiveness on the battlefield. Such swarms, inspired by natural swarms of insects, can adapt their size to the specified task and have the potential to redefine future warfare. They can operate as coordinated units of various types and sizes, achieving strategic objectives together. This evolving technology poses both a promising opportunity and a significant threat in national defence scenarios. [11]

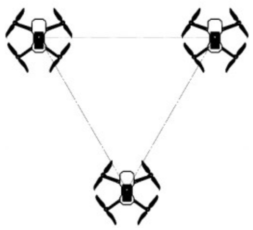


Fig. 5. Triangular swarm

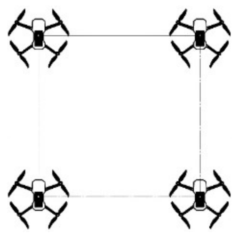


Fig. 6. Square swarm

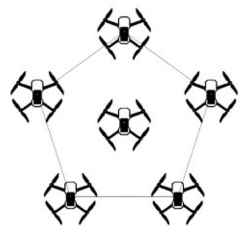


Fig. 7. Pentagon swarm

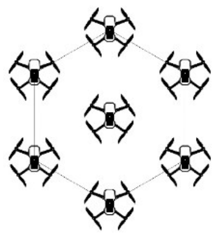


Fig. 8. Hexagon swarm

Individual drones in the swarm are programmed to form a pattern. The drones can fly in a straight line behind each other, fly in a grid or checkerboard pattern, or any other determined pattern. Assuming the use of a swarm of unmanned aerial vehicles, the appropriate grouping appears to be: Triangular (e.g. Fig. 5); Square (e.g. Fig. 6); Pentagonal (e.g. Fig. 7); Hexagonal (e.g. Fig. 8).

4. Mathematical expression of minefield density

The basic formula for calculating the minefield density is equal to [12]

$$D = \frac{m}{L}, \tag{1}$$

where: D – minefield density; m – number of mines in the minefield; L – minefield length.
This value indicates the number of mines per meter of minefield length. It is the ratio of the number of mines laid in the minefield to the actual length of the minefield established. Minefield density expresses the saturation of a section of terrain with mines and should not be confused with barrier density, which expresses the saturation of an area with barriers of various types, including non-explosive barriers. [12]

The density of the minefield is selected according to the type and effectiveness of the mines used, and the combat effectiveness of the minefield is directly dependent on its value. In view of this link, principles for the selection of minefield densities are established in accordance with the requirements for the combat effectiveness of minefields. For anti-tank minefields made of anti-track mines, a value of 0.75 – 1 is given. The second type is full-width mines. Such mines are activated by means other than pressure and are therefore more effective because their activation area (the entire vehicle profile) is increased. For full-width mines, the quoted value is set at 0.2 – 0.4. Minefields formed by anti-personnel mines are not calculated because their use, manufacture and retention are prohibited in Czechia under the Ottawa Treaty. [12], [13]
The activation zone of the mine is not included in the calculations as the shape of the mine will not affect its effectiveness.
An alternative formula

$$D = \frac{n}{a}, \tag{2}$$

where: n – number of minefield rows; a – distance between mines in the row
can also be used to determine the minefield density. [12]

In terms of practice, it has some advantages over the first formula. In particular, the mine commander knows the minefield density in advance and can make an informed determination of the distances between mines. It is rather difficult to determine the value of the distance between mines in a row, because it is only possible to enter directly into the formula if these distances between mines are always the same. If the distance between mines is different on each rows of the minefield, the average distance must be added [12]

$$\varnothing\, a = \frac{a_1 \times X_1 + a_2 \times X_2 + \cdots + a_n \times X_n}{X_1 + X_2 + \cdots + X_n}, \tag{3}$$

where: $\emptyset\, a$ – average distance between mines in a row; $a_1, a_2, a_3, \dots, a_n$ – distance between mines in each row; $X_1, X_2, X_3, \dots, X_n$ – frequency (number) of occurrence of series with the corresponding distance value.
If the mine distance in each row is different, the density of the minefield could also be determined as the sum of the densities of each row according to the formula [12]

$$D = D_1 + D_2 + \cdots + D_n = \frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}. \qquad (4)$$

If all the above formulas are used for a given minefield, the calculated values may differ slightly (irrelevant for the needs of armies). Formulas (2) and (4) consider only the actual course of the minefield and do not consider the distances of mines from the control lines (right and left boundaries of the minefield). In contrast to formula (1), it is usually the distances between the control lines (minefield length) that are decisive. Partial differences in the calculated values are particularly evident for minefields of small length. [12]

Established minefields, which are arranged, are laid to cover the most suitable area when enemy combat vehicles enter the area. It is disadvantageous to construct such a minefield in a uniform grid, where the mines form regular quadrilateral formations (square, rectangle). In a theoretical passage of enemy combat vehicles through a minefield with an angle of attack of 90° and the specified directional vector, the combat vehicles would pass between mines without activating any mines. Therefore, mines are laid in a checkerboard formation where the mines on the second row of the minefield are offset by half the length between the mines on the first row. This ensures an increased probability of enemy combat vehicles raiding the mine activation zone or directly onto the mine. Existing methods of laying arranged minefields are therefore limited strictly to simple shifts between mines in a row. The only data that can be changed are the number of mines in the row (distance between mines in the row) and distance between rows. [7], [12]

It can be seen that existing methods are limited in the design of minefields, which still represent one of the key elements of modern warfare. It is therefore necessary to continually address the modernisation of laying minefields. A much more serious problem that can be observed in the war in Ukraine is the high vulnerability and conspicuousness of the units that place these minefields. Due to the high effectiveness of minefields and their considerable spread, both sides in the conflict are aware of their dangers. This exposes the units placing the minefields to the threat of being ambushed and subsequently attacked (e.g. by artillery or attack drones).

The use of remotely deployable scatterable minefields is a promising alternative, although their uneven distribution of anti-tank mines may not fully achieve the intended effect. With the rapid advancement of UAVs, employing these drones as carriers for mine deployment is becoming an increasingly viable option. Leveraging the swarming capabilities of drones enhances their potential use in the armories of many countries. To maximize the effectiveness of UAVs in laying arranged minefields, applying mathematical methods to maintain structure and consistency in mine placement is essential.

5. Mathematical expression of minefield combat effectiveness

Minefield combat effectiveness is a value indicating the probable number of combat equipment destroyed (intercepted) or casualties inflicted. It is directly dependent on the density of the minefield, the type of mines laid, and the characteristics of the equipment engaged. Thus, if we consider that a minefield formed by swarming UAVs will be composed entirely of full-width mines, the combat effectiveness of the minefield will depend on the overall width of the vehicle and the direction of approach of the vehicle into the minefield. [12]
The formula for the combat effectiveness of a minefield is based on the theory of the probability of a phenomenon occurring at least once in n trials [12]. This probability does not consider additional data such as the probability of a target entering a particular minefield, the probability of detecting a minefield, etc.

$$P_{1,n} = 1 - (1 - P_1)^n, \qquad (5)$$

where: P_1 – probability of the target hitting a mine in the first row.
The formula is usable only under the assumption that the composition of the individual series does not change and therefore the probabilities are always the same. If we want to know the opposite phenomenon (how many combat vehicles pass through the minefield), we just need to modify the formula [12]

$$P_p = (1 - P_1)^n. \qquad (6)$$

The probability of a target striking a mine is determined by the characteristics of the anti-tank mine. Assuming that mines laid by UAVs will be full-width, the formula is equal [12]

$$P_1 = \frac{w}{a}, \qquad (7)$$

where: w – width of the combat vehicle; a – distance between mines in the row.
In case a minefield is modelled that does not have regular rows (i.e. the densities of the individual rows change), a general formula is needed to calculate the combat effectiveness of the minefield

$$\prod_{k=1}^{i-1}(1 - P_k) \times P_i,\ i = 1, 2, 3, \dots n\,. \qquad (8)$$

6. Calculations of minefields created by a swarm of drones

The initial model used a square-shaped swarm to simulate a minefield laid over a 100-meter wide area. The layout involved arranging smaller swarms sequentially without side-by-side overlap. Each smaller swarm, depicted as a square with a circle inside, ensured coverage. Simple shading indicated areas covered by two adjacent swarms, while double hatching denoted coverage by all neighboring swarms. There were six such swarms, each consisting of 9 drones equipped with an anti-tank mine, totaling 54 mines. Density calculations utilized all available formulas. The minefield had 54 mines, was 100 meters long, consisted of 6 rows, with a 12.5-meter distance between mines in a row.

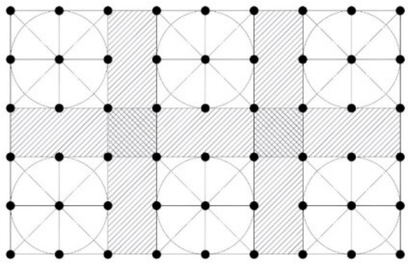


Fig. 9. A minefield made up of square swarms of UAVs

$$D = \frac{m}{L} = \frac{54}{100} = 0.54$$

Using the general density formula (1), it was calculated that the model minefield using square drone swarms reaches a density of 0.54, which exceeds the requirements for minefields consisting of full-width mines (0.3 – 0.4).

$$D = \frac{n}{a} = \frac{6}{12.5} = 0.48$$

Using the formula for determining the density of a minefield using the distances between mines and the number of rows (2), it was found that the density corresponds to a value of 0.48, which is a tolerable deviation from the previous formula, as slight differences are possible for smaller minefields

$$D = \frac{1}{12.5} \times 6 = 0.48.$$

When calculating the density of each row (4), we get the same result as in the case of the formula using the number of rows and distances between mines in the row. To calculate the combat effectiveness, it is necessary to know the width of the vehicle entering the minefield. As an example, a random battle tank with a width of 3.75 m will be given for all calculations

$$P_1 = \frac{w}{a} = \frac{3.75}{12.5} = 0.3.$$

Knowing the probability P_1 of a target hitting a mine, we can calculate the probability of traversing the entire minefield. The probability that the target (combat vehicle) hits a mine increases with each row of the minefield. For a given minefield characteristic of 6 rows, the probability of hitting a mine is 0.8824 which, when converted to a percentage, gives a rounded probability of 88.24%

$$P_{1,6} = 1 - (1 - 0.3)^6 \cong 0.8824 = 88.24\%.$$

Substituting in the modified formula gives the opposite effect, or what is the probability that a combat vehicle of width w will overcome a minefield of 6 rows. When rounded and converted to percentages, it gives 11.76%

$$P_p = (1 - P_1)^n = (1 - 0{,}3)^6 = 0.1176 = 11.76\%.$$

However, as mentioned in the paragraphs above, minefields cannot be laid in a grid without overlapping rows. The entire minefield would have to be laid at a certain angle towards the enemy. A 45° angle would ensure that the each row would form an already functioning minefield. But even a 45° angle is not entirely effective. A sharper angle of between 25° and 35° appears to be a suitable angle for a minefield grid. A perpendicular entry of the combat vehicles into the minefield would increase many times the number of mines that would be on a collision course with the combat equipment. This method is a target for future research. As in the previous case, the minefield created by triangular swarms was modelled. Due to the ideal shape, where the rows overlap and thus meet the functionality requirements. The minefield, containing 48 anti-tank mines, was modeled using 10 swarms of 6 drones each. To maintain the grid and address gaps, rotating some swarms is recommended. However, the triangular layout cannot perfectly meet the minefield requirements, particularly at the edges. These edge gaps could link to additional mine-laid areas. Density was calculated using all applicable formulas. The minefield is 100 meters long, has 6 rows, and the mines are spaced 12.5 meters apart.

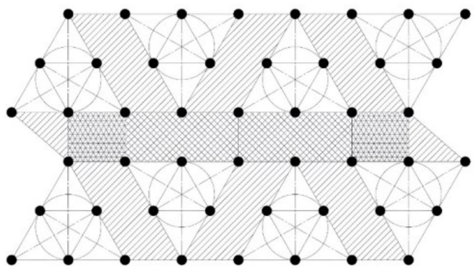


Fig. 10. A minefield made up of triangular swarms of UAVs

$$D = \frac{m}{L} = \frac{48}{100} = 0.48.$$

Using the general density formula (1), it was calculated that the model field using triangular drone swarms reaches a density of 0.48, which exceeds the requirements for minefields consisting of full-width mines (0.3 – 0.4). Using formula (2) to calculate the density yields the same result as for a minefield modelled from quarter swarms. The difference between the two models is known more in practical application, since the basic formulas assume knowledge of the basic rules for laying minefields (and hence for the principle of overlapping rows in succession)

$$D = \frac{n}{a} = \frac{6}{12.5} = 0.48.$$

Summing the densities on each row separately, the resulting value will be identical to the value calculated through formulas (1) and (2)

$$D = \frac{1}{12.5} \times 6 = 0.48.$$

When calculating the probability of hitting a target on a mine in a minefield, we use the same parameters

$$P_1 = \frac{w}{a} = \frac{3.75}{12.5} = 0.3.$$

Since the calculation of the probability of hitting a target on a mine considers only the distance of mines in the row and the total number of rows, the resulting value will be similar to the previous modelling

$$P_{1,6} = 1 - (1 - 0{,}3)^6 \cong 0.8824 = 88.24\%.$$

When substituted into the probability formula, the value settles at 88.24%. The inverse phenomenon, i.e., what is the probability that a given technique will pass through the minefield, equals 11.76%

$$P_p = (1 - P_1)^6 = (1 - 0{,}3)^6 \cong 0.1176 = 11.76\%.$$

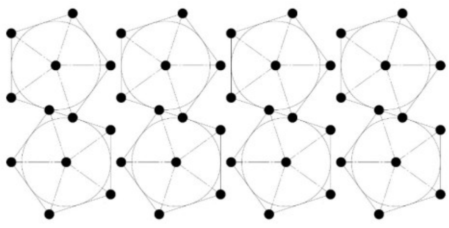
The pentagonal swarm was another model focused on minefields made of regular pentagons. In contrast to the previous modelling, it shows phenomena not observed in triangular or square grids. In fact, the distances between mines in a row will vary depending on the type of row. In total, 8 smaller swarms of 6 drones in each swarm were modelled. The first row of these swarms faced right, while the second row of swarms faced left. Using the basic density formula (1), a value of 0.48 was obtained

$$D = \frac{m}{L} = \frac{48}{100} = 0.48.$$

Since there are two rows in the pattern (third and eighth row of the minefield), which have different distances between mines, it is necessary to use the formula (3) for calculating the average distance between mines. Using this formula, the average distance between mines is 23.04 m

$$\emptyset\, a = \frac{25.6 \times 8 + 12.8 \times 2}{8 + 2} = \frac{230.4}{10} = 23.04\, m.$$

This value can already be applied to the formula for calculating the minefield density using the number of rows and distances between mines in a row (2). In this case, the measured value was equal to 0.434



$$D = \frac{n}{a} = \frac{10}{23.04} = 0.434.$$

Using the formula (4) for calculating the density of each row, a value of 0.46875 was obtained

$$D = 8 \times \frac{1}{25.6} + 2 \times \frac{1}{12.8} = 0.46875.$$

When trying to calculate the combat effectiveness of a minefield, we run into the problem of differing probabilities of hitting a mine in each row of the minefield. For this reason, it is not possible to use the basic formula for calculating combat effectiveness (5). First, the probability of hitting a mine in each row must be determined. This is obtained using formula (8), which was developed for this work. The width of the combat vehicle is the same as in the previous models (3.75 m)

$$P_n = \begin{cases} \frac{w}{a_1} = \frac{3.75}{25.6} \cong 0.1465; n = 1,2,4,5,6,7,9,10 \\ \frac{w}{a_2} = \frac{3.75}{12.8} \cong 0.2930; n = 3,8. \end{cases} \quad (9)$$

Table 1.

Combat effectiveness of minefield established by pentagon swarms

Rows - n	**Probability of hitting a mine in the row**	**Number of vehicles entering row**	**Number of vehicles entering row**	**Probability of hitting a mine in row n**	**Cumulative losses in the rows**	**Cumulative losses in the rows (%)**
Row 1	0.1465	1	100	0.1465	0.1465	14.65
Row 2	0.1465	0.8535	85.35	0.1250	0.2715	27.15
Row 3	0.2930	0.7285	72.85	0.2134	0.4849	48.49
Row 4	0.1465	0.5151	51.51	0.0754	0.5604	56.04
Row 5	0.1465	0.4396	43.96	0.0644	0.6248	62.48
Row 6	0.1465	0.3752	37.52	0.0550	0.6797	67.97
Row 7	0.1465	0.3203	32.03	0.0469	0.7267	72.67
Row 8	0.2930	0.2733	27.33	0.0801	0.8067	80.67
Row 9	0.1465	0.1933	19.33	0.0283	0.8350	83.50
Row 10	0.1465	0.1650	16.50	0.0242	0.8592	85.92

Fig. 11. A minefield made up of pentagonal swarms of UAVs

The total combat effectiveness of the minefield is in the table 1 at the end of row 10. The combat effectiveness value of the pentagonal grid is 85.92%.

Complex calculations for military use are impractical. The calculation of combat effectiveness serves mainly to inform commanders for decision-making, as predicting enemy movement in a minefield is uncertain. Simplifying this calculation is advisable by using a procedure similar to previous models (triangular and square grids). The average probability of a target hitting a mine in a row can be calculated similarly to how average minefield length is determined by summing the probabilities per row and dividing by the total number of rows

$$\emptyset P_1 = \frac{P_{n1} \times X_1 + P_{n2} \times X_2}{X_1 + X_2} = \frac{0.1465 \times 8 + 0.2930 \times 2}{8 + 2} \cong 0.1758. \quad (10)$$

To perform the necessary calculations, we obtain a value for the average combat effectiveness fixed at 0.1758. This value can already be inserted into the standard formula for calculating combat effectiveness (5)

$$P_{1,10} = 1 - (1 - 0.1758)^{10} \cong 0.8553 \cong 85.53\ \%.$$

The resulting value of combat effectiveness equals 85.53% which is almost the same value as the more complicated (but more accurate calculation – 85.95%).

Like the pentagonal grid, the hexagonal grid features rows without an orderly arrangement due to varying mine distances. The grid consists of 8 rows with 20-meter minimum distances and 5 rows with 40-meter minimum distances. For simulations, 8 groups of 7 drones each were utilized. The figure shows areas of "dead" space, with different hatching patterns indicating coverage by two, three, or four neighbouring drone swarm.

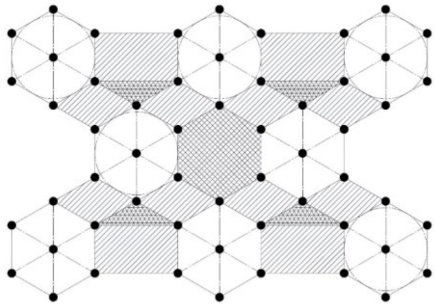


Fig. 12. A minefield made up of hexagonal swarms of UAVs

These "dead" zones would distort the total distance between the left and right boundaries of the minefield. It is therefore necessary to introduce an average minefield length. A closer examination of the figure of the model hexagonal grid shows that the first five and last five rows of the minefield have a standard length of 100 m. The middle three rows of the minefield, however, are truncated and are two mines (one on each side of the minefield) shorter in length, and therefore their total length equates to a minefield of 60 meters.
The formula for calculating the average length showed a value of 90.77 m

$$\emptyset\, L = \frac{n_1 \times L_1 + n_2 \times L_2}{n} = \frac{10 \times 100 + 3 \times 60}{13} = \\ = \frac{1180}{13} = 90.77\ m. \tag{11}$$

This value would be further used in the basic formula (1) to calculate the density using the number of mines and the length (average length) of the minefield. The resulting density value is equal to 0.617

$$D = \frac{m}{L} = \frac{56}{90.77} = 0.617.$$

As with the previous modelling, it is necessary to determine the average distance between mines, as the minefield series are not the same (3). The resulting value of the average distance between mines in a row is equal to 27.69 m

$$\emptyset\, a = \frac{40 \times 5 + 20 \times 8}{5 + 8} = \frac{360}{13} = 27.69\ m.$$

This value can be plugged into the formula (2) for calculating the minefield density

$$D = \frac{n}{a} = \frac{13}{27.69} = 0.469.$$

Using the formula for calculating the density in each row (4), we get a result of 0.525

$$D = 5 \times \frac{1}{40} + 8 \times \frac{1}{20} = 0.125 + 0.4 = 0.525.$$

The calculation of combat effectiveness is based on the same principles as the calculation of combat effectiveness for the pentagonal structure.

$$P_n = \begin{cases} \dfrac{w}{a_1} = \dfrac{3.75}{40} = 0.09375; n = 1,3,7,11,13 \\ \dfrac{w}{a_2} = \dfrac{3.75}{20} = 0.1875; n = 2,4,5,6,8,9,10,12. \end{cases}$$

Table 2.

Combat effectiveness of minefield established by hexagon swarms

Rows - n	**Probability of hitting a mine in the row**	**Number of vehicles entering row**	**Number of vehicles entering row (%)**	**Probability of hitting a mine in row n**	**Cumulative losses in the rows**	**Cumulative losses in the rows (%)**
Row 1	0.09375	1	100	0.09375	0.09375	9.38
Row 2	0.1875	0.9063	90.63	0.16992	0.2636	26.37
Row 3	0.09375	0.7363	73.63	0.06903	0.3327	33.27
Row 4	0.1875	0.6673	66.73	0.12512	0.4578	45.78
Row 5	0.1875	0.5422	54.22	0.10166	0.5595	55.95
Row 6	0.1875	0.4405	44.05	0.08260	0.6421	64.21
Row 7	0.09375	0.3579	35.79	0.03356	0.6756	67.56
Row 8	0.1875	0.3244	32.44	0.06082	0.7345	73.45

Row 9	0.1875	0.2635	26.35	0.04942	0.7859	78.59
Row 10	0.1875	0.2141	21.41	0.04015	0.8260	82.60
Row 11	0.09375	0.1740	17.40	0.01631	0.8423	84.23
Row 12	0.1875	0.1577	15.77	0.02956	0.8719	87.19
Row 13	0.09375	0.1281	12.81	0.01201	0.8839	88.39

As with the pentagonal structure of the swarm laying the minefield, we can calculate the combat effectiveness in the same way as with the previous model (10)

$$\emptyset\, P_1 = \frac{0.09375 \times 5 + 0.1875 \times 8}{5 + 8} = 0.1514.$$

To perform the necessary calculations, we obtain a value for the average combat effectiveness fixed at 0.1514. This value can already be inserted into the standard formula for calculating combat effectiveness (5)

$$P_{1,13} = 1 - (1 - 0.1514)^{10} \cong 88.17\ \%.$$

The resulting value of combat effectiveness equals 88.17% which is almost the same value as the more complicated (but more accurate calculation – 88.39%).

7. Discussion

To model minefields laid using UAV swarms, it is necessary to understand the main principles that apply to their creation. This work has brought a basic understanding to the area under discussion. The scope and application of drones is wide and will therefore be further researched. When modelling individual minefields using the specified patterns, these patterns need to be placed so that they do not overlap where a potential drone may already be. At the same time, there must not be a phenomenon where there are mines at different distances from each other in the same row of mines. Thus, within a row, it must be the case that the mines are always equidistant from each other. If this condition is met, it is possible to model multiple rows with different distances of mines from each other, but only ever within the respective row. According to the authors, it is possible that there is a pattern that may not appear orderly at first glance, but at the same time the mines are placed within them according to given parameters and not randomly as is the case with minefields laid in a scatter pattern. Such a pattern should manifest itself in patterns that are not laid in a grid but are rotated about their axis. Patterns of 7 or more points (heptagons, octagons, ...) were not used for the modelling of minefields because the more the number of sides in a circle increases, the more the shape resembles a circle.

However, it is possible that future research will consider these patterns. The primary motivation for researching unmanned mine-laying is to accurately track the positions of anti-tank mines via modern drones equipped with GPS. This capability enhances the efficiency of minefield recovery, allowing for the safe retrieval of unactivated mines post-conflict and potentially preventing civilian casualties. Additionally, the research seeks to move away from traditional minefield layouts, such as checkerboard or grid patterns, by exploring non-standard formations like pentagonal arrangements. These patterns, while precisely configured, appear irregular and help to avoid the predictability of conventional mine templating.

8. Conclusion

The results of the calculations suggest that it is possible to create minefields using UAVs in a swarm. The density values of the modelled minefields comply with the prescribed standards and also meet the requirements for their establishment. There were no marginal differences between the modelled examples. The measured deviations do not show a significant difference between the models. The contribution of this work lies in the description and outlining of the possibilities to lay minefields differently than the established procedures.

Calculations and modelling show that minefields using unmanned vehicles are a possible alternative, although their prevalence is so far low. In light of the experience of the war in Ukraine, it is certain that minelaying will become a routine part of future forces operating with unmanned vehicles. Pressure to protect units is increasing, which is spurring research in this area.

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